

## PREFERENCES DISTRIBUTIONS DENSITIES FOR A COMMON CONTINUOUS ALTERNATIVE

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*The article reveals an attempt to solve a problem of optimal control over the situation of multi-alternativeness and proneness to conflict of operational modes. Due to the research, a new approach discloses a scientific explanation and prognosis of the optimal ways of marine vessels operation and control adjusting the environmental protection support on the basis of the subjective entropy maximum principle. The attempt was made in application to the ballast water treatment. The author obtained preferences distributions densities for a common continuous alternative on the basis of a special kind functional with the effectiveness function set in a logical condition.*

**Keywords:** dangerous goods, harmful substances, ballast operations, operational effectiveness, optimal control, subjective entropy maximum principle, variational problem, Euler-Lagrange equation, preferences distribution density.

**Introduction.** A discussion on the topic of the rational modes of operation concerning a concept of ballast water treatment optimal intensity on the basis of multi-alternativeness initiated in work [1] has many applications.

**Urgency of researches.** It is an actual task to prolong such a type of researches as a kind of a process of a multi-alternative operational modes control for a few special cases of alternatives and their possible combinations. It is an important scientific problem to generalize previously obtained results.

**Analysis of the latest researches and publications.** In the previous publication [1] we have theoretically considered and analyzed the processes of the ship water ballast treatment and found the optimal value with the use of a preference function for a continuous alternative.

Similarly to that, it was considered some cases of alternatives in works [2, 3]. At the decision making, the responsible person's controlling behavior is driven by the laws of subjective conservatism [4].

Theoretical foundations are monographs [5–7].

The necessary information for the illustrative applications is adopted from the international source [8].

**The task setting.** In this paper we will be finding a continuous alternative preferences distribution, common for two discrete alternatives, each of which has a continuous alternative in the view of the common variated parameter.

**The main content (material).** The idea is to continue studying the operational modes combinations started in [1–4] in order to generalize modeling dependences of control in situations with possible alternative strategies, for example, applicably to a ship ballast water treatment. The entropy of preferences is a tool.

**The problem formulation.** Accordingly to [1–3] we have got the solution of a simplest variational problem in the view of the canonical distribution of the active system controlling individual's preferences on the basis of the subjective entropy extremization principle [4–7].

Now, let us consider a more general case with a functional with an integrand of a special kind.

Let the first alternative strategy or operational mode is given with the functional

$$\Phi_{\pi_1} = \int_{x_0}^{x_1} [-\pi_1(x) \ln \pi_1(x) - \beta_1 \pi_1(x) R_1(x)] dx + \gamma \left[ \int_{x_0}^{x_1} \pi_1(x) dx - 1 \right], \quad (1)$$

where  $[x_0 \dots x_1]$  – possible changing diapason of the continuous parameter  $x$ ;  $\pi_1(x)$  – function of the individual's subjective preferences distributed on the set of reachable for the responsible

person's goals continuous alternative related to the parameter of  $x$ ;  $\beta_1, \gamma$  – structural parameters [1–7];  $R_1(x)$  – function of effectiveness.

For the second alternative

$$\Phi_{\pi_2} = \int_{x_0}^{x_1} [-\pi_2(x) \ln \pi_2(x) - \beta_2 \pi_2(x) R_2(x)] dx + \gamma \left[ \int_{x_0}^{x_1} \pi_2(x) dx - 1 \right]. \quad (2)$$

Herewith in the expressions of (1) and (2) we imply the same possible changing diapason of the continuous parameter  $x$ . However the structural parameters  $\gamma$  may be different, although we do not distinguish them, denoting as the Lagrange uncertainty multipliers.

For the common distribution in the case of these two competing alternatives we have got the analogous view functional

$$\Phi_{\pi} = \int_{x_0}^{x_1} [-\pi(x) \ln \pi(x) - \beta \pi(x) R(x)] dx + \gamma \left[ \int_{x_0}^{x_1} \pi(x) dx - 1 \right], \quad (3)$$

but, the integrand in (3) is given with the logical equation

$$R(x) = \begin{cases} R_2(x) & \text{if } R_2(x) < R_1(x), \\ R_1(x) & \text{otherwise.} \end{cases} \quad (4)$$

The sign « $\ll$ » in combination with « $-\beta$ » at  $\beta > 0$  is for the minimal value of  $\Phi_{\pi}$ . In case of searching for the maximum, they should be changed for the opposite.

The condition (4) does not mean choosing either  $\pi_1(x)$  or  $\pi_2(x)$  depending upon the relative function of effectiveness. It will be illustrated with the problem solution that it is not just a choice of  $\pi_1$  and  $\pi_2$ .

**The problem solution.** On choosing the desired view of the operational functional (1-3) with respect to the equation (4) we get the canonical distributions of the subjective preferences applying the necessary conditions for extremums in the view of Euler-Lagrange equations for the variational problems

$$\frac{\partial R^*}{\partial \pi_i} - \frac{d}{dx} \left( \frac{\partial R^*}{\partial \pi'_{i,x}} \right) = 0, \quad (5)$$

where  $R^*$  – the under-integral function of the corresponding integral (1-3);  $\pi'_{i,x}$  – the first derivative of the corresponding sought preference function with respect to the independent variable  $x$ .

In our simplest case

$$\frac{\partial R^*}{\partial \pi'_{i,x}} \equiv 0, \quad (6)$$

therefore

$$\frac{\partial R^*}{\partial \pi_i} = 0. \quad (7)$$

Using (5-7)

$$\pi_i(x) = \frac{e^{-\beta_i R_i(x)}}{\int_{x_0}^{x_1} e^{-\beta_i R_i(x)} dx} \quad (8)$$

For the common distribution in the case of (3), the expression (8) gets the view:

$$\pi(x) = \frac{e^{-\beta R(x)}}{\int_{x_0}^{x_1} e^{-\beta R(x)} dx} \quad (9)$$

where  $R(x)$  satisfies the condition (4).

**Practical application of the problem solution.** For a practical application of the problem solution we consider, for example, a concept of [1-3].

For the ship operators, the major problem is to make optimal decisions concerning the modes of operation and effective functioning of their vessels.

According to [8] and following [1] we will consider a variation of the controlled parameter  $x$  (the intensity of the ballast water treatment) and a few, however, principal functions dependent upon the parameter. These are:  $Y(x)$  – harmful effects in total, resulting from the ballast waters organisms, their sediments, and other negative factors that accompany the processes of the ballast water treatment;  $S(x)$  – total, separately for the spent efforts, effects of the treatment implementation; and the summary result:  $R(x)$  of the both effects:  $Y(x)$  and  $S(x)$ , which on condition of a common dimension would be

$$R(x) = S(x) + Y(x). \quad (10)$$

The environmental protection engineering paradigm says that  $Y(x)$  has a general tendency for decreasing and  $S(x)$  – for increasing while the intensity of the corresponding environmental protection measures support:  $x$  grows. In such circumstances we look for the minimal value of  $R(x)$  with respect to  $x$ , which is believed to be the optimal.

Taking into account the discrete or continuous character of the parameter of  $x$ , we consider a corresponding set of operational alternatives and get the operational controlling functions in the view of canonical distributions of individual preferences [1–7] that symbolizes the effectiveness of the vessel’s functioning, thus the optimal control of her operation in the situation of multi-alternativeness.

As for a ship ballast water treatment, numerical simulation with the use of some mathematical modeling dependencies demonstrates the optimal generalized intensity of the ballast water treatment.

Supposing the continuous character of the intensity  $x$  in the possible changing diapason  $[x_0 \dots x_1]$  and the mathematical modeling dependencies similar to [1]

$$Y(x) = \frac{1}{a(x-d)}, \quad (11)$$

where  $a$  and  $d$  – parameters;

$$S(x) = b(x-d)^c, \quad (12)$$

where  $b$  and  $c$  – parameters; we obtain the model for expression (10):

$$R(x) = b(x-d)^c + \frac{1}{a(x-d)}. \quad (13)$$

In analogy to [1]-[3], we define the operational functionals (1-3) with respect to the equations (4), (10-13).

The sought individual's preferences functions will have got the canonical view (8), (9) accordingly to the considered problem setting.

It is quite easy to check that, obviously, the normalizing conditions in each problem setting are satisfied:

$$\int_{x_0}^{x_1} \pi(x) dx = 1. \tag{14}$$

**The researches results.** The calculation experiment for (1-14) with the assumed data is illustrated in fig. 1.

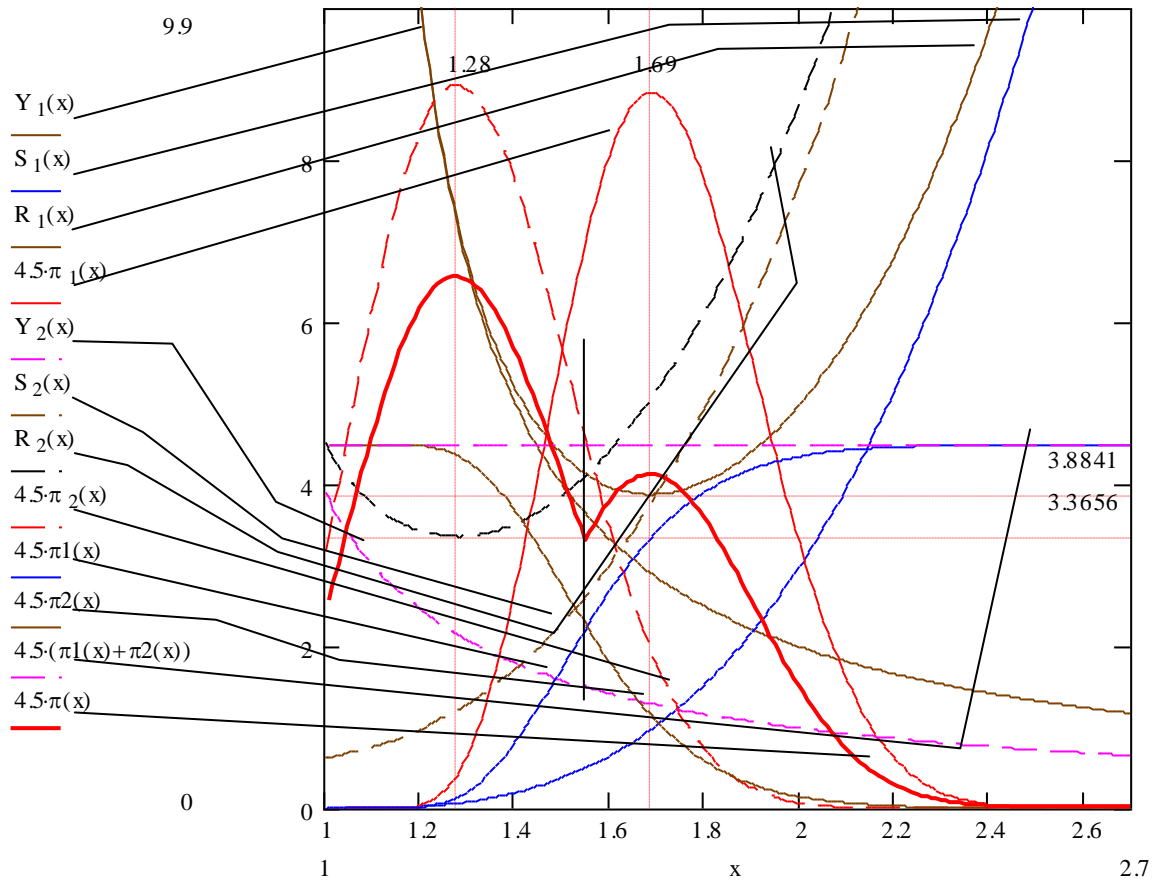


Fig. 1 – The optimal values of the operational alternatives as the intensity of the ballast water treatment and its controlling functions as the individual preferences function densities distributed on the continuous alternatives

The assumed data accepted for the operational functional (1):  $x_0=1$ ;  $x_1=3$ ;  $a=0.5$ ;  $c=3$ ;  $b=3$ ;  $d=1$ ;  $\beta=0.9$ .

For the second considered operational functional (2), there are differences in  $a=0.75$ ;  $d=0.657$ ; and

$$S(x) = 0.5 + b(x-d)^c. \tag{15}$$

From the diagrams in fig. 1 it is noticeable that for the common continuous alternative, the preference function of  $\pi(x)$  shows up a convenient shape of the normalized value with the density distribution and relative magnitudes. This is not visible with the density distributions of  $\pi_1(x)$  or  $\pi_2(x)$  only.

The other case is shown in fig. 2.

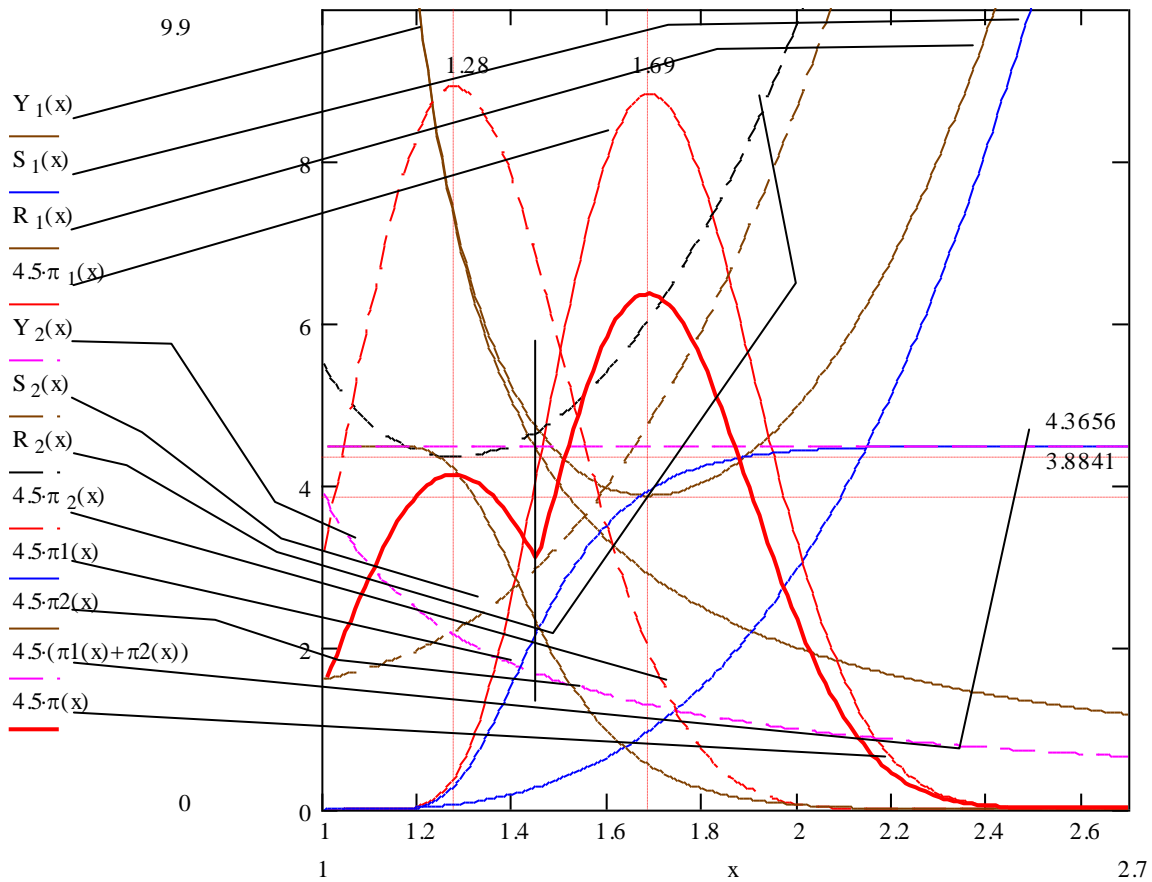


Fig. 2 – Change of the optimal values of the controlling functions as the individual preferences function densities

In the case illustrated in fig. 2 it is visible that the density distributions of  $\pi_1(x)$  and  $\pi_2(x)$  have not changed in comparison with the previous distribution. The difference was for equation (15)

$$S(x) = 1.5 + b(x - d)^c \tag{16}$$

The preferences for the discrete alternatives  $\pi_1(x)$  and  $\pi_2(x)$  show how much and which of them is better just for a certain value of the independent variable of  $x$ . The preferences distribution densities  $\pi_1(x)$  and  $\pi_2(x)$  show the optimal values of the continuous alternatives separately (see fig. 1 and fig. 2).

**Conclusions.** Due to the suggested approach, with the use of a functional compiled in a special way (3) that implies a logic condition (4), the obtained preferences distributions densities for a common continuous alternatives (9) allow carrying out a conflict free control of operational strategies and modes, for example, a ship’s ballast water treatment.

The optimal control in the considered operational situations needs a comparison of the initial discrete alternatives through their preferences functions.

**Prospects of further researches.** For further researches it is prospective to deal with the entropy paradigm in studying multi-alternativeness of operational situations on conditions of possible conflicts with the use of the methods (1)-(16) applicably to other more general cases of operational control.

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**Гончаренко А.В.** ЩІЛЬНОСТІ РОЗПОДІЛІВ ПЕРЕВАГ ДЛЯ СПІЛЬНОЇ НЕПЕРЕРВНОЇ АЛЬТЕРНАТИВИ

*Здійснено спробу розв'язати проблему оптимального керування для ситуації багатоальтернативності та конфліктності експлуатаційних режимів. У цій статті запропоновано підхід, що дає наукове пояснення та прогноз оптимальних шляхів експлуатації та керування морськими суднами із урахуванням підтримання охорони навколишнього середовища на основі принципу максимуму суб'єктивної ентропії. Спробу здійснено у застосуванні до обробки баластних вод. Отримано щільності розподілів переваг для спільної неперервної альтернативи на основі функціоналу спеціального виду із функцією ефективності, заданою через логічну умову.*

**Ключові слова:** небезпечні вантажі, шкідливі речовини, баластні операції, експлуатаційна ефективність, оптимальне керування, принцип максимуму суб'єктивної ентропії, варіаційна задача, рівняння Ейлера-Лагранжа, щільність розподілу переваг.

**Гончаренко А.В.** ПЛОТНОСТИ РАСПРЕДЕЛЕНИЙ ПРЕДПОЧТЕНИЙ ДЛЯ ОБЩЕЙ НЕПРЕРЫВНОЙ АЛЬТЕРНАТИВЫ

*Осуществлена попытка решить проблему оптимального управления для ситуации многоальтернативности и конфликтности эксплуатационных режимов. В этой статье предложен подход, который дает научное объяснение и прогноз оптимальных способов эксплуатации и управления морскими судами с учетом поддержания охраны окружающей среды на основе принципа максимума субъективной энтропии. Попытка осуществлена в приложении к обработке балластных вод. Получены плотности распределений предпочтений для общей непрерывной альтернативы на основе функционала специального вида с функцией эффективности, заданной через логическое условие.*

**Ключевые слова:** опасные грузы, вредные вещества, балластные операции, эксплуатационная эффективность, оптимальное управление, принцип максимума субъективной энтропии, вариационная задача, уравнение Эйлера-Лагранжа, плотность распределения предпочтений.

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